Sum or difference of cubes

When to use

When you have a **binomial** and both terms can be written as **perfect cubes** (something to the power of 3).

The formulas

Sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Example 1

Factor this expression fully: $8x^3 + 27y^6$

Steps	Example
Step 1: Determine what the two cubes are (these are the x^3 and y^3 in the formula)	$x^3 \rightarrow 8x^3$ $y^3 \rightarrow 27y^6$
Step 2: Find the cube roots of them (this is x and y in the formula)	$x \rightarrow 2x$ because $(2x)^3 = 8x^3$ $y \rightarrow 3y^2$ because $(3y^2)^3 = 27y^6$
Step 3: Find the squares of the x and y you found in step 2 (x^2 and y^2 in the formula)	$x^2 \rightarrow 4x^2$ because $(2x)^2 = 4x^2$ $y^2 \rightarrow 9y^4$ because $(3y^2)^2 = 9y^4$
Step 4: Put these into the correct formula (sum or difference)	Use the **sum** formula above: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ Make the substitutions shown in steps 1 to 3: $8x^3 + 27y^6 = (2x + 3y^2)(4x^2 - (2x)(3y^2) + 9y^4)$
Step 5: Check to see if you can simplify or factor any part of it *again*. The trinomial part will not be factorable, but sometimes the binomial part is. You may also be able remove extra brackets or combine like terms somewhere	Combine those two terms in the middle of 2nd bracket. $8x^3 + 27y^6 = (2x + 3y^2)(4x^2 - 6xy^2 + 9y^4)$ This is fully factored and nothing else can be combined.

Factor this expression fully: $1 - x^6y^{12}$

Steps	Example
Step 1: Determine what the two cubes are (these are the x^3 and y^3 in the formula)	$x^3 \to 1$ $y^3 \to x^6y^{12}$
Step 2: Find the cube roots of them (this is x and y in the formula)	$x \to 1$ because $1^3 = 1$ $y \to x^2y^4$ because $(x^2y^4)^3 = x^6y^{12}$
Step 3: Find the squares of the x and y you found in step 2 (x^2 and y^2 in the formula)	$x^2 \to 1$ because $1^2 = 1$ $y^2 \to x^4y^8$ because $(x^2y^4)^2 = x^4y^8$
Step 4: Put these into the correct formula (sum or difference)	Use the **difference** formula: $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ Make the substitutions shown in steps 1 to 3: $1 - x^6y^{12} = (1 - x^2y^4)(1 + 1 \cdot x^2y^4 + x^4y^8)$
Step 5: Check to see if you can simplify or factor any part of it again. The trinomial part will not be factorable, but sometimes the binomial part is. You may also be able remove extra brackets or combine like terms somewhere	The binomial part is a difference of squares, so it can still be factored more. $(1 - x^2y^4) = (1 - xy^2)(1 + xy^2)$ Now substitute this into the answer from Step 4. $1 - x^6y^{12} = (1 - xy^2)(1 + xy^2)(1 + x^2y^4 + x^4y^8)$ There is no more factoring that can be done, so this is the final answer ☺.