Unit Conversions (Non-metric)

In order to measure anything, you need units for that measurement. The problem is that there are often many different units used to measure the same thing. For example, distance might be measurement in metres, millimetres, kilometres, inches, feet, yards, miles, fathoms, palms, light-years or even beard-seconds, depending on what is being measured and who is doing the measuring. However, since all these things measure the same thing – distance – you can take a measurement in one unit and convert it to another.

Relationships between units

The only information you need in order to find in order to convert a measurement from one unit to another is a mathematical *relationship* between those units. The relationship needs to include the units you begin with and the units you want to convert to.

Here is a relationship I could use to convert between feet and metres:

$$1 \text{ metre} = 3.28 \text{ feet}$$ [1]

Most relationships are written this way, with one of something being equal to a number of something else.

The next step is to use the relationship to create a *conversion factor*.

Conversion factors

A conversion factor is a fraction (or ratio) that is equal to one, and can be multiplied by a quantity to change the unit measurement without changing the amount.

Conversion factors are built by taking a relationship like the one above, and expressing it as a fraction, with one side in the numerator and the other in the denominator.

For example, the relationship 1 metre = 3.28 feet can be written as:

$$\frac{1\, metre}{3.28\, feet} \text{ or } \frac{3.28\, feet}{1\, metre}$$

Both of these fractions are actually equal to *one* because the numerator and denominator are actually the same amount. Therefore, multiplying a quantity by either of these will not change the amount of the quantity, which is good, because all we want to change is the unit of measurement.

It will be easier to how all this works with an example.

How to do a unit conversion

The following example shows the process of converting **15 metres into feet**.

We know that 1 metre = 3.28 feet, so we can write this as a fraction two different ways:

$$\frac{1 \text{ metre}}{3.28 \text{ feet}} \quad \text{or} \quad \frac{3.28 \text{ feet}}{1 \text{ metre}}$$

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[1] Both of these conversions are technically approximations. You may see them written using the symbol ≈ (which means *approximately equal to*), instead of the = sign. Other times, you may see the symbol ≡ (which means *equals by definition*). For example, 3 feet ≡ 1 yard. In this case, the conversion is exact.

The trick is choosing which of these to multiply by. If we started out with 15 metres, which of these fractions could we multiply by in order to cancel out the metres and leave us with feet?

The answer is the second one, because it has metres on the bottom, which will cancel with the metres in the original. Here it is, step by step:

1) We start out with 15 metres, which can be rewritten as the fraction: $\frac{15 \text{ metres}}{1}$
2) Multiply this by the conversion factor $\left|\frac{3.28 \text{ feet}}{1 \text{ metre}}\right|$.[2]
$$\frac{15 \text{ metres}}{1} \times \left|\frac{3.28 \text{ feet}}{1 \text{ metres}}\right|$$
3) The metres on the top cancels with the metres on the bottom the same way numbers cancel when you multiply fractions together. The only unit left is feet, and it is in the numerator where we want it to be for the answer.
$$\frac{15 \cancel{\text{ metres}}}{1} \times \left|\frac{3.28 \text{ feet}}{1 \cancel{\text{ metres}}}\right| = \frac{15 \times 3.28 \text{ feet}}{1 \times 1} = 49.2 \text{ feet}$$
4) After cancelling the units, just work out the numbers the same way you do when multiplying fractions. In this case, 15 × 3.28 = 49.2.

Therefore, 15 metres = 49.2 feet.

Now, say you wanted to convert **100 feet into metres**. This time, use the *other* fraction above so that *feet* will cancel, and metres will be left over.

$$\frac{100 \cancel{\text{ feet}}}{1} \times \left|\frac{1 \text{ metres}}{3.28 \cancel{\text{ feet}}}\right| = \frac{100 \text{ metres}}{3.28} = 30.5 \text{ metres}$$

Therefore, 100 feet = 30.5 metres.

Multiple-step conversion problems

Sometimes you may not have a unit relationship to go straight from the given units to the required units in one step; however, there is often a way to use the relationships available to convert in multiple steps. As a guide, you can use the following three steps for problem solving in general:

1. Identify the problem
2. Develop a plan
3. Execute the plan.

Let's look at the problem of converting 60 ounces into kilograms using only the following information:

Identify the problem (what do you need to do?)

We want to convert 60 **ounces** into **kilograms**, using this information:

16 ounces = 1 pound

2.2 pounds = 1 kg

[2] It is common in textbooks to place these conversion factors in straight-line brackets like this. Mathematically, they work just like normal brackets.

Develop a plan

We don't have the information to convert 60 ounces to kilograms in one step, but we can get there in two steps by converting 16 ounces to pounds first, and then converting that number of pounds into kilograms.

$\boxed{\text{60 ounces}} \Rightarrow \boxed{\text{ ? pounds}} \Rightarrow \boxed{\text{ ? kilograms}}$

The plan goes as follows:

- Step 1: Use the relationship *16 ounces = 1 pound* to convert ounces to pounds
- Step 2: Use the relationship *2.2 pounds = 1 kilogram* to convert those pounds to kilograms

Execute the plan

Using the same method as before, this is how it would look:

Step 1: $\frac{60\ ounces}{1} \times \left|\frac{1\ pounds}{16\ ounces}\right| = 3.75 \text{ pounds}$

Step 2: $\frac{3.75 \text{ pounds}}{1} \times \left|\frac{1\ kilogram}{2.2\ pounds}\right| = 1.7 \text{ kilograms}$

Therefore, 60 ounces = 1.7 kilograms

Conversions with compound units

A compound unit is a unit of measure that is made up of a combination of other units. Examples of compound units include rates like km/h or kg/m^3, or it can be some other combination like ft·lb (foot-pounds, i.e. feet multiplied by pounds) or $kg{\cdot}m^2/s^3$.

For unit conversions with compound units, you can use a method very similar to the one above using fractions. Here is an example:

Convert 80 miles/hour into metres/second. (Both miles/hour and metres/second are measures of speed or velocity). We are given the information that there are 1609 metres in one 1 mile.

80 miles per hour is the same thing, mathematically, as 80 miles divided by 1 hour or…

$$\frac{80\ miles}{1\ hour}$$

Writing it this way will make the conversion easier.

First, I want to change the miles to metres using the information that 1609 metres = 1 mile.

I can multiply $\frac{80\ miles}{1\ hour}$ by either $\left|\frac{1609\ metres}{1\ mile}\right|$ or $\left|\frac{1\ mile}{1609\ metres}\right|$.

If I want to cancel out miles in the original fraction and replace it with metres, I'll need multiply by the first of these because it has *miles* on the bottom.

This is the result:

$$\frac{80\ \cancel{\text{miles}}}{1 \text{ hour}} \times \left|\frac{1609\ metres}{1\ \cancel{mile}}\right| = \frac{80 \times 1609 \text{ metres}}{1 \text{ hour}}$$

Without multiplying the numbers, you can see that the miles units will cancel and we'll be left with metres/hour as the result. Unfortunately, we were asked to convert to metres/second. Therefore, we need another conversion factor to convert hours to seconds.

There are 60 seconds in a minute and 60 minutes in an hour, so the number of seconds in an hour is 60×60=3600.

If 1 hour = 3600 seconds, then I can multiply by either $\left|\frac{1\ hour}{3600\ seconds}\right|$ or $\left|\frac{3600\ seconds}{1\ hour}\right|$.

Since seconds was on the bottom in the original amount $\left(\frac{80\ miles}{1\ hour}\right)$ I should use the one with hours on the top, so they will cancel each other out.

However, instead of multiplying by one conversion factor at a time, I'm going to try and do it all in one step!! I can take the original $\frac{80\ miles}{1\ hour}$ and multiply by both factors at the same time!

$$\frac{80\ miles}{1\ hour} \times \left|\frac{1609\ metres}{1\ mile}\right| \times \left|\frac{1\ hour}{3600\ seconds}\right|$$

$$= \frac{80\ \cancel{miles}}{1\ \cancel{hour}} \times \left|\frac{1609\ metres}{1\ \cancel{mile}}\right| \times \left|\frac{1\ \cancel{hour}}{3600\ seconds}\right|$$ ....After cancelling

$$= \frac{80 \times 1609}{3600}\left(\frac{metres}{seconds}\right)$$ ....After multiplying the fractions and separating the units from the numbers.

$$= 35.8\left(\frac{metres}{seconds}\right)$$

Therefore, 80 miles/hour is the same as 35.8 metres/second.

Practice Questions

Question 1:

Convert 20 feet into metres.

Question 2:

Convert 100 miles/hour into metres/second.

Question 3:

Convert 8.31 N·m/mol·K to ft·lbf/lbmol·R, given that:

1 N = 0.2248 lbf

1 m = 3.28 ft

1 K = 1.8 R

1 mol = 453.59 lbmol

Answers: (Q1: 6.1 m, Q2: 29.7 m/s, Q3: 0.00229 (or 2.29×10^{-3}) ft·lbf/lbmol·R)