Thermodynamic Systems

And Performance Measurement

Contents

Thermodynamic Systems

With an understanding of the concepts of work, energy and power, it is now possible to look at actual systems that receive energy from the outside world and use it to do mechanical work.

A thermodynamics system is defined as "a precisely defined region of the universe that is studied using the principles of thermodynamics."[1]

Defining the System

Thermodynamic analysis won't make any sense unless you define the *boundaries* of the system under consideration. Anything within the boundaries is part of the system, and everything outside the boundaries is part of the surroundings.

The First Law

The first law of thermodynamics basically says that energy can be changed from one form to another (e.g. converting kinetic to electric potential energy), but cannot be created or destroyed.

> *There is a fact, or if you wish, a law, governing all natural phenomena that are known to date. There is no known exception to this law—it is exact so far as we know. The law is called the conservation of energy. It states that there is a certain quantity, which we call energy that does not change in manifold changes which nature undergoes. That is a most abstract idea, because it is a mathematical principle. It says that there is a numerical quantity which does not change when something happens. It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number and when we finish watching nature go through her tricks and calculate the number again, it is the same.*
>
> —The Feynman Lectures on Physics

An important variation of the First Law that is useful for our purposes is what we call the Energy Balance Equation:

$$E_2 - E_1 = Q - W$$

Where: E_2 is the final amount of energy (at the end) of the process

E_1 is the initial amount of energy

Q is the net amount of heat transferred into the system from its surroundings (Q will be negative if heat was transferred to the surroundings)

W is the net work done by the system to its surroundings (W will be negative if work was done by the surroundings to the system)

[1] http://en.wikipedia.org/wiki/Thermodynamic_system

Thermodynamic Efficiency

The efficiency of a thermodynamic process is a ratio of the useful output to the input, both measured in terms of energy. It is usually shown as

$$\eta = \frac{P_{out}}{P_{in}},$$

where η (lowercase Greek letter nu) represents *thermodynamic efficiency*, P_{out} is the amount of useful output power (e.g. electric power, mechanical work done per unit time, or heat output per unit time) and P_{in} is the amount of power (energy per unit time) that is input to the system. Output power should never be more than input power, so efficiency will never be greater than 1.

Example of Thermodynamic Efficiency

Imagine an electric motor that uses 50 kJ of electricity to pump 50 kg of water from a well 30 m underground to the surface. With this information, we can figure out the efficiency.

Efficiency is also equal to W_{out}/W_{in} just so long as the period of time is the same for both.

$P_{in} = \frac{W_{in}}{t}$ and $P_{out} = \frac{W_{out}}{t}$; therefore,

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\frac{W_{out}}{t}}{\frac{W_{in}}{t}} = \frac{W_{out}}{W_{in}}$$

This is why we don't need to know the time it takes to complete the action.

W_{in} = 50kJ, the amount of energy inputted to the system.

To find W_{out}, use the formula for gravitational potential energy.

$$W_{out} = PE_f - PE_i$$

$$W_{out} = mgz_f - mgz_i$$

$$W_{out} = 50\text{kg} \times 9.8\text{m/s}^2 \times 0\text{m - }50\text{kg} \times 9.8\text{m/s}^2 \times (\text{-}30\text{m})$$

$$W_{out} = 0 - (-14700\text{J})$$

$$W_{out} = 14700\text{J}$$

$$W_{out} = 14.7\text{kJ}$$

Now we just plug W_{in} and W_{out} into the equation for efficiency we derived at the beginning of the question.

$$\eta = \frac{W_{out}}{W_{in}}$$

$$\eta = \frac{14.7kJ}{50kJ}$$

$$\eta = 0.294$$

Other Kinds of Performance Measurement

In some cases, other measures of performance might be more appropriate. Sometimes these measures are given other names to separate them from "thermodynamic efficiency", defined above. You need to be careful when using the word "efficiency", which has a precise definition. That is why terms like "coefficient of performance" are used for other measures of performance.

One thing all these measures have in common is that they all follow the general formula:

$$\text{Performance Measurement} = \frac{\text{Desired Effect}}{\text{Cost}}$$

If this was a business course, the performance measurement might be called "productivity", the desired effect could be the number of units produced, and the cost could be total amount of money spent to produce those units, including the cost of labour, materials and overhead.

Since this is an engineering-related course, the desired effect is usually something like the amount of power generated, or it could be a different effect altogether, like an amount of heat produced (if you're trying to keep your home warm in the winter) or heat expelled (if you're trying to keep your home cool in the summer).

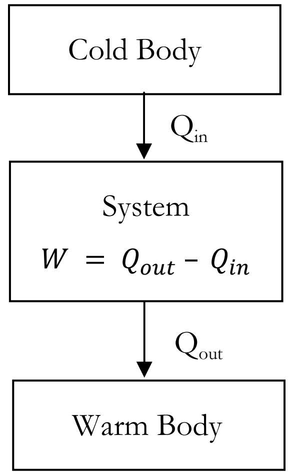
For heat pumps and refrigerators, it is helpful to look at something called the *coefficient of performance*. It has a slightly different definition for heat pumps vs. refrigerators, but both of them follow the general idea of performance measurement being equal to desired effect divided by cost.

For both heat pumps and refrigerators, we can also use this handy equation:

$$W_{cycle} = Q_{out} - Q_{in}$$

This equation is true because work, defined as the energy transferred from the system to the surroundings, will be, in the case of a heat pump or refrigerator, the energy transferred from the cold body to the warm body (going through the system in between). This is why work can be calculated the same way for both heat pump and refrigeration cycles.

The figure below describes both heat pumps and refrigerators. The only difference between the two is the *desired effect*. Specifically, for a heat pump, the desired effect is the Q_{out}, and for a refrigerator, the desired effect is the Q_{in}. This will be explained the next two sections.



Refrigerator

A refrigerator or air conditioner takes heat from the environment and absorbs it into the system, thus making the environment cooler, and then it expels the heat into a hotter environment. According to the 2nd Law of Thermodynamics, this kind of heat interaction never occurs spontaneously; instead, work has to be done to make it happen.

For a refrigeration system, the coefficient of performance is calculated with this formula.

$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$\beta = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

Where: β is the coefficient of performance – note that (unlike η) β can be greater than 1.
Q_{in} is the amount of heat inputted to the system from the cold body
W_{cycle} is the amount of work done to the system to accomplish this
Q_{out} is the amount of heat outputted from the system to the surroundings (the warm body).

The desired effect is heat going into the system from the environment (making the environment cooler), and this is represented by Q_{in}. The cost is the amount of work needed to accomplish this.

Heat Pump

A heat pump transfers heat from a cold environment to the system to a warm environment. The desired effect is heat being transferred to the warm environment (although the heat also needs to be transferred from a colder body first). The warm environment might be your house, and the system consists of the machine that does the work, and the colder body is the outside world. Thus, as with the refrigerator, it takes heat from a colder body (the outside) and transfers it to a warmer one (the inside of the house). Again, this cannot happen unless the machinery in the heat pump does work to make it happen somehow.

The coefficient of performance for a heat pump is calculated in a similar way as a refrigerator:

$$\gamma = \frac{Q_{out}}{W_{cycle}}$$

$$\gamma = \frac{Q_{out}}{Q_{out} - Q_{in}}$$

This time, the desired effect is to output heat from the system to the environment, which is represented by Q_{out}. Again, the cost is the amount of work required.

Again, note that γ (lowercase Greek letter gamma) can be greater than 1 (and in fact, will always be greater than or equal to 1).

Power Cycle

A power cycle takes inputted heat and transforms it into work done to the surroundings. We use the term *thermal efficiency* to measure the efficiency of this type of process.

Energy balance applies to this situation, but since all forms of energy should be unchanged at the end of a completed cycle, all we are left with is:

$W_{cycle} = Q_{cycle}$ or

$W_{cycle} = Q_{in} - Q_{out}$ (note this is the reverse of the equation for the refrigeration/heat pump cycle)

We can also find equations for thermal efficiency:

$$\eta = \frac{W_{cycle}}{Q_{in}} \quad \text{or}$$

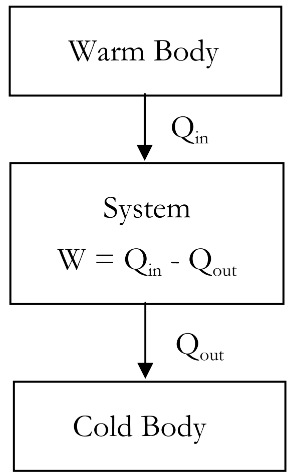
$$\eta = \frac{Q_{in} - Q_{out}}{Q_{in}} \quad \text{or}$$

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

Thermal efficiency is very similar to thermodynamic efficiency, and must always be less than 1 (i.e. $\eta < 1$).

Example:

For a power cycle operating as in the figure below, the heat transfers are Q_{in} = 25,000 kJ and Q_{out} = 15,000 kJ. Determine the net work, in kJ, and the thermal efficiency.



Answer:

$$\begin{aligned} W_{cycle} &= Q_{in} - Q_{out} \\ &= 25000kJ - 15000kJ \\ &= 10000kJ \end{aligned}$$

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

$$\eta = 1 - \frac{15000}{25000}$$

$$\eta = 0.4$$

Finding work done by gas expansion process

A lot of gas expansion problems involve the property pV^n = Constant. What does that mean? Mostly, it means you can solve for p and V values at different points in the process using the equation:

$$p_1V_1^n = p_2V_2^n$$

Which is true because both $p_1V_1^n$ and $p_2V_2^n$ are equal to the same constant.

Here, p_1 and V_1 represent the initial pressure and volume for the system and p_2 and V_2 represent the pressure and volume at another point in the process, e.g. after an expansion or compression.

The value of n depends on the gas involved, but will also vary depending on other factors like temperature and the amount of heat transfer in the process.

For an adiabatic process where the gas is air, n = 1.4

For a system with constant pressure, n = 0

Formulas for finding work

For n=1, $W = p_1V_1\ln\left(\frac{V_2}{V_1}\right)$

For n=0, $W = p(V_2 - V_1)$

For all other values of n, $W = \frac{p_2V_2 - p_1V_1}{1-n}$

W will be positive for expansion and negative for compression.

How to solve for n:

You can solve for the value of n as long as you know two pairs of values for p and V. To solve for any unknown inside an exponent, you need to know about *logarithms*.

Example: For a gas expansion, pressure is initially 10 bar and volume is 0.5 m^3. After the expansion, the volume is 1 m^3 and the pressure is 4 bar. If the relationship between pressure and volume is given by pV^n = Constant, then what is the value of n? Also, what is the work for the process in kJ?

Answer: $p_1 = 10$ bar $V_1 = 0.5\ m^3$

$P_2 = 4$ bar $V_2 = 1\ m^3$

$$p_1V_1^n = p_2V_2^n$$

$$\frac{V_1^n}{V_2^n} = \frac{p_2}{p_1}$$ Move both the 'n' terms on the same side.

$$\left(\frac{V_1}{V_2}\right)^n = \frac{p_2}{p_1}$$ Combine into one exponent for the whole left side.

$$\ln\left(\frac{V_1}{V_2}\right)^n = \ln\frac{p_2}{p_1}$$ Take the natural logarithm of both sides to bring n out of the exponent.

$$n\ln\frac{V_1}{V_2} = \ln\frac{p_2}{p_1}$$

$$n = \frac{\ln\frac{p_2}{p_1}}{\ln\frac{V_1}{V_2}}$$ Isolate n.

$$n = \frac{\ln\frac{4bar}{10bar}}{\ln\frac{.5m^3}{1m^3}}$$ Substitute the values for known variables.

$$n = \frac{\ln\frac{4}{10}}{\ln\frac{.5}{1}}$$ Compute the value of n.

$$n = \frac{\ln 0.4}{\ln 0.5}$$

$$n = 1.32$$

Now, let's find the work. For n=1.32, use $W = \frac{p_2V_2 - p_1V_1}{1-n}$

To do that, convert the pressure values from bars to Pa, because 1 Pa times 1 m^3 will equal 1 J, which we can convert to kJ at the end.

$$W = \frac{p_2V_2 - p_1V_1}{1-n}$$

$$W = \frac{(400000\ Pa)(1\ m^3) - (1000000\ Pa)(0.5\ m^3)}{1-1.32}$$

$$W = \frac{400000J - 500000J}{-0.32}$$

$$W = \frac{-100000}{-0.32}$$

$$W = 312500\ J$$

$$W = 312.5\ kJ$$

Therefore, the amount of work for the process is 312.5 kJ.

Putting it together – energy balance in closed systems

By combining the First Law of Thermodynamics with the concepts of work, heat transfer and energy, we can use the following equation for closed systems.

$$E_2 - E_1 = Q - W$$

Including the different forms of energy for a closed system, we can change this to:

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Where: ΔKE is the change in kinetic energy

ΔPE is the change in potential energy

ΔU is the change in internal energy

Q is the amount of heat transferred into the system (negative if heat is going out of the system).

W is the amount of work done by the system to its surroundings (negative if work is done to the system by surroundings).

Example:

A closed system of mass 4 kg undergoes a process in which there is heat transfer of 40 kJ from the system to the surroundings. The system's elevation increases by 700m during the process. The specific internal energy of the system decreases by 20 kJ/kg and the system also somehow goes through a horizontal velocity change from 2 m/s to 10 m/s. The acceleration due to gravity is 9.7 m/s^2. What is the work, in kJ?

Answer:

Given:

m = 4kg

Q = -40kJ

Δz = 700m…use to find ΔPE!

Δu = -20kJ/kg

v_i=2m/s

v_f=10m/s…use to find ΔKE!!

Trying to Find:

W=?? kJ

Solution:

ΔPE+ΔKE+ΔU=Q – W

ΔU and Q are known. Need to find missing ΔPE and ΔKE first in order to find W.

$$\begin{aligned}\Delta PE &= mg\Delta z\\ &= (4kg)\left(\frac{9.7m}{s^2}\right)(700m)\\ &= 27160kg\cdot\frac{m^2}{s^2}\\ &= 27160J = 27.16kJ\end{aligned}$$

$$\begin{aligned}\Delta KE &= \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2\\ &= \frac{1}{2}m\left({v_f}^2 - {v_i}^2\right)\\ &= \frac{1}{2}(4kg)\left[\left(\frac{10m}{s}\right)^2 - \left(\frac{2m}{s}\right)^2\right]\\ &= 2kg\left(100\frac{m^2}{s^2} - 4\frac{m^2}{s^2}\right)\\ &= 192kg\frac{m^2}{s^2}\\ &= 192J = 0.192kJ\end{aligned}$$

Specific internal energy is internal energy per unit mass. Need to multiply this amount by the mass to find the internal energy.

$$\begin{aligned}\Delta U &= \Delta u\cdot m\\ &= -20\frac{J}{kg}\cdot 4\,kg\\ &= -80\,J\end{aligned}$$

Now we can plug everything into the Energy Balance equation.

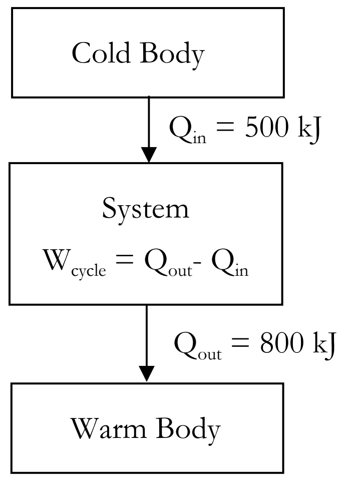
$$\begin{aligned}\Delta KE + \Delta PE + \Delta U &= Q - W\\ W &= Q - (\Delta KE + \Delta PE + \Delta U)\\ W &= 40kJ - (0.192kJ + 27.160kJ - 20\text{kJ})\\ W &= 32.648kJ\end{aligned}$$

Therefore the work for the process is 32.648 kJ.

Problems

Heat pump questions

1) Calculate the coefficient of performance for the heat pump cycle described in the diagram below.

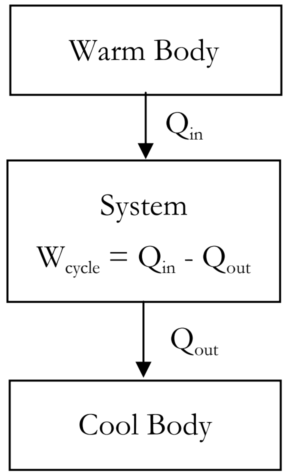


Refrigerator questions

2) Calculate the coefficient of performance for a refrigeration cycle where Q_{out} = 35000kJ and W_{cycle} = 10000 kJ.

Power cycle questions

3) For a power cycle operating as in the figure below, W_{cycle} = 800 Btu and Q_{out} = 1800 Btu. What is the thermal efficiency?



4) The thermal efficiency of a power cycle operating as shown in the above figure is 30%, and Q_{out} = 650 MJ. Determine the net work developed and the heat transfer Q_{in}, each in MJ.

Energy balance questions

5) A closed system with mass 8 kg goes through a process during which there is heat transfer of 250 kJ from the system to the surroundings and work done on the system equals 125 kJ. The initial specific internal energy of the system is initially 725 kJ/kg. What is the final specific internal energy, in kJ/kg? Assume any changes in kinetic and potential energy are negligible.

6) A gas expands in a piston-cylinder assembly from an initial volume of 0.025 m^3 and initial pressure of 7.5 bar to a final pressure of 3.1 bar. The relation between pressure and volume is given by $pV^{1.3}$=constant. The mass of the gas in the cylinder is 0.242 kg. The specific internal energy of the gas decreases by 35.2 kJ/kg during the process. What is the work in kJ and the heat transfer in kJ?

Solutions

Question 1 (Heat Pump)

Given:

$Q_{in} = 500\ kJ$

$Q_{out} = 800\ kJ$

$\gamma = ?$

Equation:

$$\gamma = \frac{Q_{out}}{Q_{out} - Q_{in}}$$
$$\gamma = \frac{800\ kJ}{800\ kJ - 300\ kJ}$$
$$\gamma = \frac{800\ kJ}{300\ kJ}$$
$$\gamma = 2.67\ kJ$$

Therefore, the coefficient of performance for the heat pump cycle is 2.67.

Question 2 (Refrigerator)

Given:

$Q_{in} = 35000\ kJ$

$W_{cycle} = 10000\ kJ$

$\beta = ?$

Equation:

$$\beta = \frac{Q_{in}}{W_{cycle}}$$
$$\beta = \frac{35000\ kJ}{10000\ kJ}$$
$$\beta = 3.5$$

Therefore the coefficient of performance for the refrigeration cycle is 3.5.

Question 3 (Power cycle)

Given:

$W_{cycle} = 800\ BTU$

$Q_{out} = 1800\ BTU$

$\eta = ?$

Equations:

$$\eta = \frac{W_{cycle}}{Q_{in}}$$

But Q_{in} is unknown. Need to find Q_{in} before finding η.

$$W_{cycle} = Q_{in} - Q_{out}$$
$$Q_{in} = W_{cycle} + Q_{out}$$
$$Q_{in} = 800\ BTU + 1800\ BTU$$
$$Q_{in} = 2600\ BTU$$

Now use this value in the equation for η.

$$\eta = \frac{W_{cycle}}{Q_{in}}$$
$$\eta = \frac{800\ BTU}{2600\ BTU}$$
$$\eta = 0.307$$

Therefore the thermal efficiency of the power cycle is 0.307.

Question 4 (Power cycle)

Given:

$\eta = 30\% = 0.3$

$Q_{out} = 650\ MJ$

$W_{cycle} = ?$

$Q_{in} = ?$

Equation:

$$\eta = \frac{W_{cycle}}{Q_{in}} \rightarrow \text{eq'n } (1)$$

$$W_{cycle} = Q_{in} - Q_{out} \rightarrow \text{eq'n } (2)$$

We have two equations with two unknowns (W_{cycle} and Q_{in}). Solve by substituting eq'n (2) into eq'n (1), and then solving for Q_{in}.

$$\eta = \frac{Q_{in} - Q_{out}}{Q_{in}}$$
$$Q_{in}\eta = Q_{in} - Q_{out}$$
$$Q_{in}\eta - Q_{in} = -Q_{out}$$
$$Q_{in}(\eta - 1) = -Q_{out}$$
$$Q_{in} = \frac{-Q_{out}}{(\eta - 1)}$$
$$Q_{in} = \frac{-650\ MJ}{(0.3 - 1)}$$
$$Q_{in} = \frac{-650\ MJ}{-0.7}$$
$$Q_{in} = 929\ MJ$$

Now substitute this value into eq'n (2) to find W_{cycle}.

$$W_{cycle} = Q_{in} - Q_{out}$$
$$W_{cycle} = 929\ MJ - 650\ MJ$$
$$W_{cycle} = 279\ MJ$$

Therefore the net work is 279 MJ and the heat transfer Q_{in} is 929 MJ.

Question 5 (Energy balance)

Given:

$m = 8\ kg$

$Q = -250\ kJ$

$W = -125\ kJ$

$\Delta KE = 0$

$\Delta PE = 0$

$u_i = 725 \frac{kJ}{kg}$

$u_f = ?$

Equations:

$$\Delta KE + \Delta PE + \Delta U = Q - W$$
$$0 + 0 + U_f - U_i = -250\ kJ - (-125\ kJ)$$
$$U_f - U_i = -125\ kJ$$

Specific internal energy (u) is total internal energy (U) divided by mass (m). Therefore:

$$U_i = u_i \times m$$
$$U_i = 725\ \frac{kJ}{kg} \times 8\ kg$$
$$U_i = 5800\ kJ$$

Now, plug this back in the energy balance equal above.

$$U_f - U_i = -125\ kJ$$
$$U_f = -125\ kJ + U_i$$
$$U_f = -125 + 5800\ kJ$$
$$U_f = 5675\ kJ$$

Finally, we just need to divide by the mass to change this to specific internal energy.

$$u_f = \frac{U_f}{m}$$
$$u_f = \frac{5675\ kJ}{8\ kg}$$
$$u_f = 709 \frac{kJ}{kg}$$

Therefore, the final specific internal energy is 709 kJ/kg.

Question 6 (Energy balance)

Given:

$V_1 = 0.025\ m^3, V_2 = ?$

$p_1 = 7.5\ bar, \quad p_2 = 3.1\ bar$

$n = 1.3$ (The exponent of V in the pressure-volume relation.)

$m = 0.242\ kg$

$\Delta u = -35.2 \frac{kJ}{kg}$

$\Delta KE = 0$

$\Delta PE = 0$

$W = ?$

$Q = ?$

Equations:

The plan is to use the pressure-volume relation ($p_1 V_1^{1.3} = p_2 V_2^{1.3}$) to find the missing V_2 value, then use the work formula ($W = \frac{p_2 V_2 - p_1 V_1}{1-n}$) to find the work. Lastly, find the heat transfer using the energy balance equation ($\Delta KE + \Delta PE + \Delta U = Q - W$).

Step 1: Find V_2

$$p_1 V_1^{1.3} = p_2 V_2^{1.3}$$
$$V_2^{1.3} = \frac{p_1 V_1^{1.3}}{p_2}$$
$$V_2^{1.3} = \frac{(7.5\ bar)(0.025\ m^3)^{1.3}}{3.1\ bar}$$
$$V_2^{1.3} = 0.02$$
$$V_2 = \sqrt[1.3]{0.02}$$
$$V_2 = 0.049\ m^3$$

Step2: Find W

$$W = \frac{p_2 V_2 - p_1 V_1}{1-n}$$
$$W = \frac{(7.5\ bar)(0.049\ m^3) - (3.1\ bar)(0.025\ m^3)}{1-1.3}$$

Need to convert bars to Pascals for this equation to work!

$$W = \frac{(7.5 \times 10^5\ Pa)(0.049\ m^3) - (3.1 \times 10^5\ Pa)(0.025\ m^3)}{1-1.3}$$
$$W = \frac{115190 - 18750}{-0.3}$$
$$W = 11867\ J \approx 11.9\ kJ$$

Step 3: Find Q

$$\Delta KE + \Delta PE + \Delta U = Q + W$$

Since $\Delta U = \Delta u \times m$, we get

$$0 + 0 + \Delta u \times m = Q - W$$
$$-35.2 \frac{kJ}{kg} \times 0.242\ kg = Q - 11.9\ kJ$$
$$-8.52\ kJ = Q - 11.9\ kJ$$
$$Q = -8.52\ kJ + 11.9\ kJ$$
$$Q = 3.38\ kJ$$

Therefore, the work done by the gas expansion is 11.9 kJ and the heat transfer is 3.38 kJ into the system.