Place Value (and the decimal system)

In the past, there were many different methods of representing numbers (e.g. Roman numerals). Today, most of the world uses a *place value* system based on powers of 10.

For example: 5280 => 5 thousands, 2 hundreds, 8 tens, and 0 units

Another way of writing it would be:

$$5280 = 5\times1000 + 2\times100 + 8\times10 + 0\times1$$

When we read whole numbers like the own above, we understand that the 5 actually represents five thousand, the 2 represents two hundred, and so on, because of these numbers' positions relative to an imaginary reference point on the right side of the "units" digit.

When we want to represent *fractions* using the same type of positional notation, we use a **decimal point**. In North America and the United Kingdom, the decimal point is represented by a period (.), while in continental Europe, and some other parts of the world, it is represented by a comma (,). The decimal point separates the numbers that represent whole amounts from the ones that represent fractional amounts.

For example: 32.174 => 3 tens, 1 unit, 1 tenth, 7 hundredths, and 4 thousandths.

Another way of writing it would be:

$$32.174 = (3 \times 10) + (2 \times 1) + \left(1 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right)$$

You can see from this that the three digits to the right of the decimal all represent *fractions* (that is, quantities smaller than one).

Rounding

Sometimes when you are working with decimal numbers, there are too many digits in your answer and you need to make the answer shorter by *rounding it off*. Before rounding, you need to decide how many decimal places (digits to the right of the decimal) you want to keep.

For example, 32.174, rounded to two decimal places, would be 32.17. You can see that the result only has two digits to the right of the decimal. In this case, I am *rounding down*, because the result after rounding is a lower number than we had before.

The number 153.378, rounded to two decimal places, would be 153.38. In this case, we are *rounding up*, because the result after rounding is a higher number than we had before.

"Rounding to two decimal places" is another way of saying "rounding to the nearest hundredth". Can you see why? This is because the place to the right of the decimal is the *hundredths* place. In

both examples, we changed the given number to closest number to it that only goes as far as the hundredths place.

However, in order to round to the nearest hundredth, we need to look at the digit in the next place over, the thousandths place, to see whether it would be appropriate to round up or down. The general rule for rounding to a particular place is:

- If the next number to the right is a 5, 6, 7, 8, or a 9, then **round up**
- If the next number to the right is a 0, 1, 2, 3, or a 4, then **round down**

So, in the first example above (32.174 rounded to two decimal places), we rounded to the nearest hundredth. The 7 was the number in the hundredths spot. In order to determine whether to round up or down, we needed to look at the next digit to the right. Since this digit was a four, we rounded down to 32.17.

In the second example (153.378), we rounded up the 7, in the hundredths place, to an 8 because the digit to the right was 8.

Have a look at the following flow chart that describes the act of rounding.